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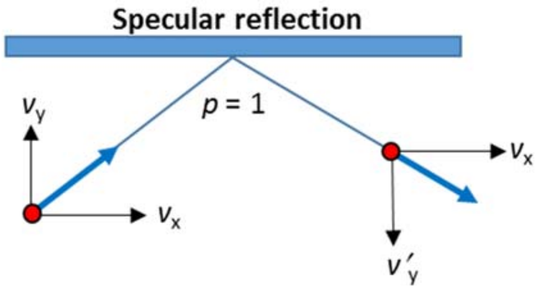
# Chapter 2

## Answers to "Why?" in the text

### Page 187, footnote 21

Figure below shows specular reflection, that is, a totally elastic collision of an electron with the surface of a film. If this were a rubber ball bouncing off a wall, then there would only be a change in the $y$-component $v_y$ of the velocity, which would be reversed. The $x$-component is unchanged. The collision has no effect on the $v_x$ component of the velocity. If there is an electric field in the $-x$ direction then the electron can continue to gain velocity from the field as if it never collided with the wall. Specular reflection does not increase the resistivity.



### Page 196, footnote 21

"Pure Al suffers badly from electromigration problems and is usually alloyed with small amounts of Cu, called Al(Cu), to reduce electromigration to a tolerable level. But the resistivity increases. (Why?)" The increase is due to Matthiessen's rule. The added impurities (Cu) in Al provide an additional scattering mechanism.

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**2.1 Electrical conduction** Na is a monovalent metal (BCC) with a density of 0.9712 g cm$^{-3}$. Its atomic mass is 22.99 g mol$^{-1}$. The drift mobility of electrons in Na is 53 cm$^2$ V$^{-1}$ s$^{-1}$.

*a.* Consider the collection of conduction electrons in the solid. If each Na atom donates one electron to the electron sea, *estimate* the mean separation between the electrons. (Note: if $n$ is the concentration of particles, then the particles' mean separation $d = 1/n^{1/3}$.)

*b.* Estimate the mean separation between an electron ($e^-$) and a metal ion ($Na^+$), assuming that most of the time the electron prefers to be between two neighboring $Na^+$ ions. What is the approximate Coulombic interaction energy (in eV) between an electron and an $Na^+$ ion?

*c.* How does this electron/metal-ion interaction energy compare with the average thermal energy per particle, according to the kinetic molecular theory of matter? Do you expect the kinetic molecular theory to be applicable to the conduction electrons in Na? If the mean electron/metal-ion interaction energy is of the same order of magnitude as the mean *KE* of the electrons, what is the mean speed of electrons in Na? Why should the mean kinetic energy be comparable to the mean electron/metal-ion interaction energy?

*d.* Calculate the electrical conductivity of Na and compare this with the experimental value of $2.1 \times 10^7\ \Omega^{-1}\ \mathrm{m}^{-1}$ and comment on the difference.

## Solution

***a.*** If $D$ is the density, $M_{\mathrm{at}}$ is the atomic mass and $N_A$ is Avogadro's number, then the atomic concentration $n_{\mathrm{at}}$ is

$$n_{\mathrm{at}} = \frac{DN_A}{M_{\mathrm{at}}} = \frac{(971.2\,\mathrm{kg\,m^{-3}})(6.022\times10^{23}\,\mathrm{mol^{-1}})}{(22.99\times10^{-3}\,\mathrm{kg\,mol^{-1}})} = 2.544\times10^{28}\,\mathrm{m^{-3}}$$

which is also the electron concentration, given that each Na atom contributes 1 conduction electron.

If $d$ is the mean separation between the electrons then $d$ and $n_{at}$ are related by (see Chapter 1 Solutions, Q1.11; this is only an estimate)

$$d \approx \frac{1}{n_{\mathrm{at}}^{1/3}} = \frac{1}{(2.544\times10^{28}\,\mathrm{m^{-3}})^{1/3}} = \mathbf{3.40 \times 10^{-10}\ m} \text{ or } \mathbf{0.34\ nm}$$

***b.*** Na is BCC with 2 atoms in the unit cell. So if $a$ is the lattice constant (side of the cubic unit cell), the density is given by

$$D = \frac{\text{(atoms in unit cell)(mass of 1 atom)}}{\text{volume of unit cell}} = \frac{2\left(\dfrac{M_{\mathrm{at}}}{N_A}\right)}{a^3}$$

isolate for $a$, $a = \left[\dfrac{2M_{\mathrm{at}}}{DN_A}\right]^{1/3} = \left[\dfrac{2(22.99\times10^{-3}\,\mathrm{kg\ mol^{-1}})}{(0.9712\times10^{3}\,\mathrm{kg\ m^{-3}})(6.022\times10^{23}\,\mathrm{mol^{-1}})}\right]^{1/3}$

so that $a = 4.284 \times 10^{-10}$ m or 0.4284 nm

For the BCC structure, the radius of the metal ion $R$ and the lattice parameter $a$ are related by $(4R)^2 = 3a^2$, so that,

$$R = (1/4)\sqrt{(3a^2)} = 1.855 \times 10^{-10}\ \text{m or } 0.1855\ \text{nm}$$

If the electron is somewhere roughly between two metal ions, then the mean electron to metal ion separation $d_{\text{electron-ion}}$ is roughly $R$. If $d_{\text{electron-ion}} \approx R$, the electrostatic potential energy $PE$ between a conduction electron and <u>one</u> metal ion is then

$$PE = \frac{(-e)(+e)}{4\pi\varepsilon_0 d_{\text{electron–ion}}} = \frac{(-1.602\times10^{-19}\,\mathrm{C})(+1.602\times10^{-19}\,\mathrm{C})}{4\pi(8.854\times10^{-12}\,\mathrm{F\,m^{-1}})(1.855\times10^{-10}\,\mathrm{m})} \quad (1)$$

∴ $\boldsymbol{PE = -1.24 \times 10^{-18}}$ **J** or **−7.76 eV**

***c.*** This electron-ion $PE$ is much larger than the average thermal energy expected from the kinetic theory for a collection of "free" particles, that is $E_{\mathrm{average}} = KE_{\mathrm{average}} = 3(kT/2) \approx 0.039$ eV at 300 K. In the case of Na, the electron-ion interaction is very strong so we cannot assume that the electrons are moving around freely as if in the case of free gas particles in a cylinder. If we assume that the mean $KE$ is roughly the same order of magnitude as the mean $PE$,

$$KE_{\text{average}} = \frac{1}{2}m_e u^2 \approx |PE|_{\text{average}} = -1.24\times10^{-18}\,\text{J} \tag{2}$$

where $u$ is the mean speed (strictly, $u$ = root mean square velocity) and $m_e$ is the electron mass.

Thus, $$u \approx \left[\frac{2PE_{average}}{m_e}\right]^{1/2} = \left[\frac{2(1.24\times10^{-18}\text{ J})}{(9.109\times10^{-31}\text{ kg})}\right]^{1/2} \tag{3}$$

so that **$u = 1.65 \times 10^6$ m/s**

There is a theorem in classical physics called the *Virial theorem* which states that if the interactions between particles in a system obey the inverse square law (as in Coulombic interactions) then the magnitude of the mean *KE* is equal to the magnitude of the mean *PE*. The Virial Theorem states that:

$$KE_{\text{average}} = -\frac{1}{2}PE_{\text{average}}$$

Indeed, using this expression in Eqn. (2), we would find that $u = 1.05 \times 10^6$ m/s. If the conduction electrons were moving around freely and obeying the kinetic theory, then we would expect $(1/2)m_e u^2 = (3/2)kT$ and $u = 1.1 \times 10^5$ m/s, a much lower mean speed. Further, kinetic theory predicts that $u$ increases as $T^{1/2}$ whereas according to Eqns. (1) and (2), $u$ is insensitive to the temperature. The experimental linear dependence between the resistivity $\rho$ and the absolute temperature $T$ for most metals (non-magnetic) can only be explained by taking $u$ = constant as implied by Eqns. (1) and (2).

***d.*** If $\mu$ is the drift mobility of the conduction electrons and $n$ is their concentration, then the electrical conductivity of Na is $\sigma = en\mu$. Assuming that each Na atom donates one conduction electron ($n = n_{at}$), we have

$$\sigma = en\mu = (1.602\times10^{-19}\text{C})(2.544\times10^{28}\text{m}^{-3})(53\times10^{-4}\text{m}^2\ \text{V}^{-1}\ \text{s}^{-1})$$

*i.e.* **$\sigma = 2.16 \times 10^7\ \Omega^{-1}\ \text{m}^{-1}$**

which is quite close to the experimental value.

*Nota Bene*: If one takes the $\text{Na}^+$-$\text{Na}^+$ separation $2R$ to be roughly the mean electron-electron separation then this is 0.37 nm and close to $d = 1/(n^{1/3}) = 0.34$ nm. In any event, all calculations are only approximate to highlight the main point. The interaction *PE* is substantial compared with the mean thermal energy and we cannot use $(3/2)kT$ for the mean *KE*!

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**2.2 Electrical conduction** The resistivity of aluminum at 25 °C has been measured to be $2.72 \times 10^{-8}$ Ω m. The thermal coefficient of resistivity of aluminum at 0 °C is $4.29 \times 10^{-3}$ K$^{-1}$. Aluminum has a valency of 3, a density of 2.70 g cm$^{-3}$, and an atomic mass of 27.

*a.* Calculate the resistivity of aluminum at −40ºC.

*b.* What is the thermal coefficient of resistivity at −40ºC?

*c.* Estimate the mean free time between collisions for the conduction electrons in aluminum at 25 °C, and hence estimate their drift mobility.

*d.* If the mean speed of the conduction electrons is about $2 \times 10^6$ m s$^{-1}$, calculate the mean free path and compare this with the interatomic separation in Al (Al is FCC). What should be the thickness of an Al film that is deposited on an IC chip such that its resistivity is the same as that of bulk Al?

*e.* What is the percentage change in the power loss due to Joule heating of the aluminum wire when the temperature drops from 25 °C to −40 ºC?

## Solution

***a.*** Apply the equation for temperature dependence of resistivity, $\rho(T) = \rho_o[1 + \alpha_o(T - T_o)]$. We have the temperature coefficient of resistivity, $\alpha_o$, at $T_o$ where $T_o$ is the reference temperature. We can either work in K or °C inasmuch as only temperature changes are involved. The two given reference temperatures are 0 °C or 25 °C, depending on choice. Taking $T_o$ = 0 °C,

$$\rho(-40°\text{C}) = \rho_o[1 + \alpha_o(-40°\text{C} - 0°\text{C})]$$

$$\rho(25°\text{C}) = \rho_o[1 + \alpha_o(25°\text{C} - 0°\text{C})]$$

Divide the above two equations to eliminate $\rho_o$,

$$\rho(-40°\text{C})/\rho(25°\text{C}) = [1 + \alpha_o(-40°\text{C})] / [1 + \alpha_o(25°\text{C})]$$

Next, substitute the given values $\rho(25°\text{C}) = 2.72 \times 10^{-8}$ Ω m and $\alpha_o = 4.29 \times 10^{-3}$ K$^{-1}$ to obtain

$$\rho(-40\text{ °C}) = (2.72\times10^{-8}\ \Omega\text{ m})\frac{[1+(4.29\times10^{-3})(-40)]}{[1+(4.29\times10^{-3})(25)]} = \mathbf{2.035 \times 10^{-8}\ \Omega\ m}$$

***b.*** In $\rho(T) = \rho_o[1 + \alpha_o(T - T_o)]$ we have $\alpha_o$ at $T_o$ where $T_o$ is the reference temperature, for example, 0° C or 25 °C depending on choice. We will choose $T_o$ to be first at 0 °C and then at −40 °C (= $T_2$) so that the resistivity at $T_2$ and then at $T_o$ are:

At $T_2$, $\rho_2 = \rho_o[1 + \alpha_o(T_2 - T_o)]$; the reference being $T_o$ and $\rho_o$ which defines $\alpha_o$

and at $T_o$ $\rho_o = \rho_2[1 + \alpha_2(T_o - T_2)]$; the reference being $T_2$ and $\rho_2$ which defines $\alpha_2$

Rearranging the above two equations we find

$$\alpha_2 = \alpha_o / [1 + (T_2 - T_o)\alpha_o]$$

*i.e.* $\alpha_{-40} = (4.29 \times 10^{-3}) / [1 + (-40 - 0)(4.29 \times 10^{-3})] = \mathbf{5.18 \times 10^{-3}\ °C^{-1}}$

<u>Alternatively</u>, consider the definition of $\alpha_2$ that is $\alpha_{-40}$

From $$\alpha_o = \frac{1}{\rho_o}\left[\frac{d\rho}{dT}\right]_{T_o}$$

we have $\alpha_{-40} = \{1/[\rho(-40\text{ °C})]\} \times \{[\rho(25°\text{C}) - \rho(-40°\text{C})] / [(25°\text{C}) - \rho(-40°\text{C})]\}$

∴ $\alpha_{-40} = 1 / [(2.035 \times 10^{-8})] \times \{(2.72 \times 10^{-8}) - (2.035 \times 10^{-8})] / [(25) - (-40)]\}$

∴ $\boldsymbol{\alpha_{-40} = 5.18 \times 10^{-3}\ \mathrm{K}^{-1}}$

***c.*** We know that $1/\rho = \sigma = en\mu$ where $\sigma$ is the electrical conductivity, $e$ is the electron charge, and $\mu$ is the electron drift mobility. We also know that $\mu = e\tau / m_e$, where $\tau$ is the mean free time between electron collisions and $m_e$ is the electron mass. Therefore,

$$1/\rho = e^2 n\tau/m_e$$

$$\therefore \qquad \tau = m_e/\rho e^2 n \qquad (1)$$

Here $n$ is the number of conduction electrons per unit volume. But, from the density $d$ and atomic mass $M_{\text{at}}$, atomic concentration of Al is

$$n_{\text{Al}} = \frac{N_A d}{M_{\text{at}}} = \frac{(6.022\times10^{23}\ \text{mol}^{-1})(2700\ \text{kg/m}^3)}{(0.027\ \text{kg/mol})} = 6.022\times10^{28}\ \text{m}^{-3}$$

so that $n = 3n_{\text{Al}} = 1.807 \times 10^{29}\ \text{m}^{-3}$

assuming that each Al atom contributes 3 "free" conduction electrons to the metal and substituting into (1),

$$\tau = \frac{m_e}{\rho e^2 n} = \frac{(9.109\times10^{-31}\ \text{kg})}{(2.72\times10^{-8}\ \Omega\,\text{m})(1.602\times10^{-19}\ \text{C})^2(1.807\times10^{29}\ \text{m}^{-3})}$$

$$\therefore \qquad \boldsymbol{\tau = 7.22 \times 10^{-15}\ \text{s}}$$

(Note: If you do not convert to meters and instead use centimeters you will not get the correct answer because seconds is an SI unit.)

The relation between the drift mobility $\mu_d$ and the mean free time is given by Equation 2.5, so that

$$\mu_d = \frac{e\tau}{m_e} = \frac{(1.602\times10^{-19}\text{C})(7.22\times10^{-15}\text{s})}{(9.109\times10^{-31}\text{kg})}$$

$$\therefore \qquad \boldsymbol{\mu_d = 1.27 \times 10^{-3}\ \text{m}^2\ \text{V}^{-1}\text{s}^{-1} = 12.7\ \text{cm}^2\ \text{V}^{-1}\text{s}^{-1}}$$

***d.*** The mean free path is $l = u\tau$, where $u$ is the mean speed. With $u \approx 2 \times 10^6\ \text{m s}^{-1}$ we find the mean free path:

$$\boldsymbol{l} = u\tau = (2 \times 10^6\ \text{m s}^{-1})(7.22 \times 10^{-15}\ \text{s}) \approx \boldsymbol{1.44 \times 10^{-8}\ \text{m} \approx 14.4\ \text{nm}}$$

A thin film of Al must have a much greater thickness than $l$ to show bulk behavior. Otherwise, scattering from the surfaces will increase the resistivity by virtue of Matthiessen's rule.

***e.*** Power $P = I^2R$ and is proportional to the resistivity $\rho$, assuming the rms current level stays relatively constant. Then we have

$$[P(-40\ °\text{C}) - P(25\ °\text{C})] / P(25\ °\text{C}) = P(-40\ °\text{C}) / P(25\ °\text{C}) - 1 = \rho(-40\ °\text{C}) / \rho(25\ °\text{C}) - 1$$

$$= (2.03 \times 10^{-8}\ \Omega\ \text{m} / 2.72 \times 10^{-8}\ \Omega\text{m}) - 1 = \mathbf{-0.254}, \text{ or } \mathbf{-25.4\%}$$

(Negative sign means a reduction in the power loss).

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**2.3 Conduction in gold** Gold is in the same group as Cu and Ag. Assuming that each Au atom donates one conduction electron, calculate the drift mobility of the electrons in gold at 22° C. What is the mean free path of the conduction electrons if their mean speed is $1.4 \times 10^6\ \text{m s}^{-1}$? (Use $\rho_o$ and $\alpha_o$ in Table 2.1.)

## Solution

The drift mobility of electrons can be obtained by using the conductivity relation $\sigma = en\mu_d$.

| $N_A$ | 6E+23 | | | q = | 1.6E-19 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | g/mole | g /cm$^3$ | 1/m$^3$ | EXPERIMENT | 1/m$^3$ | | | nΩ m | m$^2$/Vs |
| Valency | Metal | Mat | Density | nat | RH x 1E-11 | n from RH | *x* | Difference (%) | Resistivity | Hall mobility |
| 1 | Li | 6.94 | 0.54 | 4.686E+28 | -15 | 4.161E+28 | 0.89 | 11.2 | 92.8 | 16.16 |
| 1 | Na | 22.99 | 0.968 | 2.536E+28 | -24.8 | 2.517E+28 | 0.99 | 0.7 | 48.8 | 50.82 |
| 1 | K | 39.10 | 0.862 | 1.328E+28 | -42.8 | 1.458E+28 | 1.10 | -9.8 | 73.9 | 57.92 |
| 1 | Cs | 132.91 | 1.93 | 8.745E+27 | -73.3 | 8.516E+27 | 0.97 | 2.6 | 208 | 35.24 |
| 1 | Ag | 107.87 | 10.49 | 5.856E+28 | -9 | 6.936E+28 | 1.18 | -18.4 | 16.7 | 53.89 |
| 1 | Cu | 63.55 | 8.96 | 8.491E+28 | -5.4 | 1.156E+29 | 1.36 | -36.1 | 17.1 | 31.58 |
| 1 | Au | 196.97 | 19.3 | 5.901E+28 | -7.2 | 8.670E+28 | 1.47 | -46.9 | 22.6 | 31.86 |
| 2 | Mg | 24.31 | 1.74 | 4.311E+28 | -8.3 | 7.521E+28 | 1.74 | 25.6 | 44.8 | 18.53 |
| 2 | Zn | 65.38 | 7.13 | 6.567E+28 | 10.4 | -6.002E+28 | -0.91 | 291.4 | 60.1 | 17.30 |
| 2 | Ca | 40.08 | 1.54 | 2.314E+28 | -17.8 | 3.507E+28 | 1.52 | 48.5 | 33.6 | 52.98 |
| 3 | Al | 26.98 | 2.7 | 6.026E+28 | -3.4 | 1.836E+29 | 3.05 | -4.7 | 27.1 | 12.55 |
| 3 | In | 114.82 | 7.31 | 3.834E+28 | -7 | 8.917E+28 | 2.33 | 67.4 | 83.7 | 8.36 |
| | | | | | | | | | | |

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**2.15 The Hall effect** Consider a rectangular sample, a metal or an *n*-type semiconductor, with a length *L*, width *W*, and thickness *D*. A current *I* is passed along *L*, perpendicular to the cross-sectional area *WD*. The face $W \times L$ is exposed to a magnetic field density *B*. A voltmeter is connected across the width, as shown in Figure 2.40, to read the Hall voltage $V_H$.

*a*. Show that the Hall voltage recorded by the voltmeter is

$$V_H = \frac{IB}{Den} \qquad \textit{Hall voltage}$$

*b*. Consider a 1-micron-thick strip of gold layer on an insulating substrate that is a candidate for a Hall probe sensor. If the current through the film is maintained at constant 100 mA, what is the magnetic field that can be recorded per μV of Hall voltage?

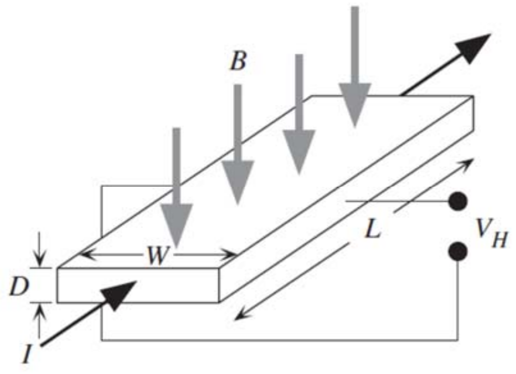

**Figure 2.40** Hall effect in a rectangular material with length *L*, width *W*, and thickness *D*. The voltmeter is across the width *W*.

## Solution

***a.*** The Hall coefficient, $R_H$, is related to the electron concentration, $n$, by $R_H = -1 / (en)$, and is defined by $R_H = E_y / (JB)$, where $E_y$ is the electric field in the $y$-direction, $J$ is the current density and $B$ is the magnetic field. Equating these two equations:

$$-\frac{1}{en} = \frac{E_y}{JB}$$

$$\therefore \qquad E_y = -\frac{JB}{en}$$

This electric field is in the opposite direction of the Hall field ($E_H$) and therefore:

$$E_H = -E_y = \frac{JB}{en} \qquad (1)$$

The current density perpendicular (going through) the plane $W \times D$ (width by depth) is:

$$J = \frac{I}{WD}$$

$$\therefore \qquad W = \frac{I}{JD} \qquad (2)$$

The Hall voltage ($V_H$) across $W$ is:

$$V_H = WE_H$$

If we substitute expressions (1) and (2) into this equation, the following will be obtained:

$$V_H = \frac{IB}{Den}$$

Note: this expression only depends on the thickness and not on the length of the sample.

In general, the Hall voltage will depend on the specimen shape. In the elementary treatment here, the current flow lines were assumed to be nearly parallel from one end to the other end of the sample. In an irregularly shaped sample, one has to consider the current flow lines. However, if the specimen thickness is uniform, it is then possible to carry out meaningful Hall effect measurements using the van der Pauw technique as discussed in advanced textbooks.

***b.*** We are given the depth of the film $D$ = 1 micron = 1 μm and the current through the film $I$ = 100 mA = 0.1 A. The Hall voltage can be taken to be $V_H$ = 1 μV, since we are looking for the magnetic field $B$ per μV of Hall voltage. To be able to use the equation for Hall voltage in part (*a*), we must find the electron concentration of gold. Appendix *B* in the textbook contains values for gold's atomic mass ($M_{\text{at}}$ =196.97 g mol$^{-1}$) and density ($d$ = 19.3 g/cm$^3$ = 19300 kg/m$^3$). Since gold has a valency of 1 electron, the concentration of free electrons is equal to the concentration of Au atoms.

$$\therefore \qquad n = \frac{dN_A}{M_{at}} = \frac{(19300\text{ kg m}^{-3})(6.022\times10^{23}\text{ mol}^{-1})}{(196.97\times10^{-3}\text{ kg mol}^{-1})} = 5.901\times10^{28}\text{ m}^{-3}$$

Now the magnetic field $B$ can be found by using the equation for the Hall voltage:

$$V_H = \frac{IB}{Den}$$

$$\therefore \qquad B = \frac{V_H D e n}{I} = \frac{\left(1\times10^{-6}\ \text{V}\right)\left(1\times10^{-6}\ \text{m}\right)\left(1.602\times10^{-19}\ \text{C}\right)\left(5.901\times10^{28}\ \text{m}^{-3}\right)}{\left(0.1\,\text{A}\right)}$$

$\therefore$ **$B$ = 0.0945 T**

As a side note, the power ($P$) dissipated in the film could be found very easily. Using the value for resistivity of Au at $T$ = 273 K, $\rho$ = 22.8 nΩ m, the resistance of the film is:

$$R = \frac{\rho L}{A} = \frac{\rho L}{WD} = \frac{\left(22.8\times10^{-9}\ \Omega\,\text{m}\right)\left(0.001\,\text{m}\right)}{\left(0.0001\,\text{m}\right)\left(1\times10^{-6}\ \text{m}\right)} = 0.228\,\Omega$$

The power dissipated is then:

$$P = I^2R = (0.1\ \text{A})^2(0.228\ \Omega) = 0.00228\ \text{W}$$

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**2.16 Electrical and thermal conductivity of In** Electron drift mobility in indium has been measured to be 6 cm2 $\text{V}^{-1}\ \text{s}^{-1}$. The room temperature (27 °C) resistivity of In is $8.37\times10^{-8}$ Ωm, and its atomic mass and density are 114.82 amu or g $\text{mol}^{-1}$ and 7.31 g $\text{cm}^{-3}$, respectively.

*a.* Based on the resistivity value, determine how many free electrons are donated by each In atom in the crystal. How does this compare with the position of In in the Periodic Table (Group IIIB)?

*b.* If the mean speed of conduction electrons in In is $1.74\times10^{8}$ cm $\text{s}^{-1}$, what is the mean free path?

*c.* Calculate the thermal conductivity of In. How does this compare with the experimental value of 81.6 W $\text{m}^{-1}\ \text{K}^{-1}$?

## Solution

***a.*** From $\sigma = en\mu_d$ ($\sigma$ is the conductivity of the metal, $e$ is the electron charge, and $\mu_d$ is the electron drift mobility) we can calculate the concentration of conduction electrons ($n$):

$$n = \frac{\sigma}{e\mu_d} = \frac{(8.37\times10^{-8}\ \Omega\ \text{m})^{-1}}{(1.602\times10^{-19}\ \text{C})(6\times10^{-4}\ \text{m}^2\ \text{V}^{-1}\ \text{s}^{-1})}$$

*i.e.* $n = 1.243\times10^{29}\ \text{m}^{-3}$

Atomic concentration $n_{\text{at}}$ is

$$n_{\text{at}} = \frac{dN_A}{M_{\text{at}}} = \frac{(7.31\times10^{3}\ \text{kg m}^{-3})(6.022\times10^{23}\ \text{mol}^{-1})}{(114.82\times10^{-3}\ \text{kg mol}^{-1})}$$

*i.e.* $n_{\text{at}} = 3.834\times10^{28}\ \text{m}^{-3}$

Effective number of conduction electrons donated per In atom ($n_{\text{eff}}$) is:

$$n_{\text{eff}} = n / n_{\text{at}} = (1.243\times10^{29}\ \text{m}^{-3}) / (3.834\times10^{28}\ \text{m}^{-3}) = \mathbf{3.24}$$

Conclusion: Within the classical theory of metals, this would imply that about three electrons per atom are donated to the conduction-electron sea in the metal. This is in good agreement with the position of the In element in the Periodic Table (III) and its valency of 3.

***b.*** If $\tau$ is the mean scattering time of the conduction electrons, then from $\mu_d = e\tau/m_e$ ($m_e$ = electron mass) we have:

$$\tau = \frac{\mu_d m_e}{e} = \frac{(6\times10^{-4}\ \text{m}^2\ \text{V}^{-1}\ \text{s}^{-1})(9.109\times10^{-31}\ \text{kg})}{(1.602\times10^{-19}\ \text{C})} = 3.412 \times 10^{-15}\ \text{s}$$

Taking the mean speed $u \approx 1.74 \times 10^6$ m s$^{-1}$, the mean free path ($l$) is given by

$$l = u\tau = (1.74 \times 10^6\ \text{m s}^{-1})(3.412 \times 10^{-15}\ \text{s}) = 5.94 \times 10^{-9}\ \text{m or } 5.94\ \text{nm}$$

One can estimate the interatomic separation $d$ from

$$d \approx \frac{1}{n_{\text{at}}^{1/3}} = \frac{1}{(3.83\times10^{28}\ \text{m}^{-3})^{1/3}} = 0.3\ \text{nm}$$

which means that $l = 20d$. The electrons passes ~20 atoms before it is scattered.

***c.*** From the Wiedemann-Franz-Lorenz law, thermal conductivity is given as:

$$\kappa = \sigma T C_{\text{WFL}} = (8.37 \times 10^{-8}\ \Omega\ \text{m})^{-1}(27\ ^{\circ}\text{C} + 273\ \text{K})(2.44 \times 10^{-8}\ \text{W}\ \Omega\ \text{K}^{-2})$$

*i.e.* $\kappa = 85.4$ W m$^{-1}$ K$^{-1}$

This value reasonably agrees with the experimental value.

*Note*: Indium has a body-centered tetragonal crystal structure and the lattice constants are $a = b = 0.325$ nm and $c = 0.494$ nm. The atomic concentration is therefore $n_{\text{at}} = 2/abc = 3.83 \times 10^{28}$ m$^{-3}$, which is the same as $n_{\text{at}} = dN_A/M_{\text{at}}$ (as we know from Ch. 1).

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**2.17 Electrical and thermal conductivity of Ag** The electron drift mobility in silver has been measured to be 54 cm$^2$ V$^{-1}$ s$^{-1}$ at 27 °C. The atomic mass and density of Ag are given as 107.87 amu or g mol$^{-1}$ and 10.50 g cm$^{-3}$, respectively.

*a.* Assuming that each Ag atom contributes one conduction electron, calculate the resistivity of Ag at 27 °C. Compare this value with the measured value of $1.6 \times 10^{-8}$ Ω m at the same temperature and suggest reasons for the difference.

*b.* Calculate the thermal conductivity of silver at 27 °C and at 0 °C.

## Solution

***a.*** Atomic concentration $n_{\text{at}}$ is

$$n_{\text{at}} = \frac{dN_A}{M_{\text{at}}} = \frac{(10.50\times10^3\ \text{kg m}^{-3})(6.022\times10^{23}\ \text{mol}^{-1})}{(107.87\times10^{-3}\ \text{kg mol}^{-1})} = 5.862 \times 10^{28}\ \text{m}^{-3}$$

If we assume there is one conduction electron per Ag atom, the concentration of conduction electrons ($n$) is $5.862 \times 10^{28}$ m$^{-3}$, and the conductivity is therefore:

$$\sigma = en\mu_d = (1.602 \times 10^{-19}\ \text{C})(5.862 \times 10^{28}\ \text{m}^{-3})(54 \times 10^{-4}\ \text{m}^2\ \text{V}^{-1}\text{s}^{-1}) = 5.071 \times 10^7\ \Omega^{-1}\ \text{m}^{-1}$$

and the resistivity, $\rho = 1/\sigma = 19.7$ nΩ m

The experimental value of $\rho$ is 16 nΩ m. We assumed that exactly 1 "free" electron per Ag atom contributes to conduction. This may not necessarily be true.

*Note:* More importantly, the difference is part of the failure of classical physics. Some of this will be apparent in Ch. 4 where energy bands are used for conduction.